Review Final Exam MAC 1114 (Trigonometry)

Show your work. Justify each answer Do not use a calculator

(1) a) Draw each angle and convert to radians: $\theta = 330^o$, $\theta = -225^o$

b) Draw each angle and convert to degrees: $\theta = \frac{-2\pi}{3}$, $\theta = 3$

c) Find the length of the arc of a circle of radius r subtended by a central angle of θ when $r = 3$ meters, $\theta = 120^o$

d) Find the area of the sector of a circle of radius 6 feet formed by a central angle of $\theta = 36^o$

e) $r = 5$ miles, arc length $s = 3$ miles find θ

(2) $\sin\theta = \frac{1}{3}$ find the exact value of each of the remaining five trigonometric functions of the acute angle θ

(3) a) A right triangle has a hypotenuse of length 8 inches. If one angle is 30^o, find the length of each leg.

(4) Find the exact value of a) $6\tan 45^o - 8\cos 60^o$;

b) $\cot 40^o - \frac{\sin 50^o}{\sin 40^o}$ c) $1 - \cos^2 20^o - \cos^2 70^o$ d) Find the exact value of $\sin^2 30^o + \cos^2 60^o$; $1 + \tan^2 30^o - \csc^2 45^o$

(5) If $\sin\theta < 0$ and $\tan\theta > 0$, in what quadrant does θ lie?

(6) Find the reference angle a) -20^o, b) 240^o; c) -120^o; d) 70^o; e) $\frac{5\pi}{7}$

(7) Find the exact value of $\cos(210^o)$; $\tan(120^o)$; $\sec(420^o)$, $\sin(9\pi/4)$, $\sin(-240^o)$, $\tan(14\pi/3)$, $\cot(-\pi/6)$

(8) If $\sin\theta = \frac{-5}{13}$ and θ in quadrant III. Find the exact value of each of the six trigonometric functions of θ

(9) If $\tan\theta = \frac{-2}{3}$ and θ in quadrant II. Use **trigonometric identities** to find the exact value of $\cos\theta$; $\sin\theta$

(10) Find the domain, range and graph one cycle of each trig. function

a) $f(x) = \sin x$? b) $f(x) = \cos x$? c) $f(x) = \tan x$?

d) $f(x) = \sec x$?

e) $f(x) = \cot x$? f) $f(x) = \csc x$?

(11) Suppose that you are headed toward a plateau 35 feet high. If the angle of elevation to the top of the plateau is 30^o, how far are you from the base of the plateau?

(12) a) Determine the amplitude, period of $y = \frac{3}{2} - \frac{1}{2}\sin(\frac{\pi}{8}x)$. Be sure to label the 5 key points and sketch two cycles.

(13) Determine the period of a) $y = \tan(x/2)$ b) $y = -1 + 2\cot(x)$ c) $y = \csc(3\pi x/2)$ d) $y = 1 + 3\sec(x/4)$

(14) State the definition

a) $\cos^{-1} x$ b) $\csc^{-1} x$ c) $\sin^{-1} x$ d) $\tan^{-1} x$ e) $\sec^{-1} x$ f) $\cot^{-1} x$

(15) Find the exact value of each expression (Justify each answer)

a) $\sin^{-1}(-1)$ b) $\cos^{-1}(\frac{-\sqrt{2}}{2})$ c) $\sin^{-1}(\frac{-1}{2})$ d) $\cos^{-1}(\frac{-1}{2})$ $\cos^{-1}(-1)$

e) $\sec^{-1}(-\sqrt{2})$ f) $\tan^{-1}(1)$ g) $\cot^{-1}(-\sqrt{3})$ h) $\csc^{-1}(\frac{-2}{\sqrt{3}})$ $\tan^{-1}(-\sqrt{3})$

(16) Find the exact value of each expression (Justify each answer), if there is no value, explain why

a) $\sin^{-1}(\sin(\frac{3\pi}{8}))$ b) $\cos^{-1}(\cos(\frac{3\pi}{4}))$ c) $\tan^{-1}(\tan(\frac{-2\pi}{5}))$ d) $\sin^{-1}(\sin(\frac{-8\pi}{9}))$ $\sin(\sin^{-1}(-0.7))$

e) $\cos(\cos^{-1}(\sqrt{2}))$ f) $\tan(\cos^{-1}(\frac{-4}{5}))$ g) $\cos(\csc^{-1}(\frac{5}{3}))$ h) $\sin(\cot^{-1}(\frac{3}{4}))$ $\cos(\tan^{-1}(\frac{-4\pi}{3}))$

(17) a) State the sum formulas for cosine, sine, tangent

b) State the double-angle formulas for cosine, sine, tangent

c) State the half-angle formulas for cosine, sine

d) State 3 Product-to-sum formulas

(18) Use the sum or difference formulas. Do not use your calculator.

a) $\sin\alpha = \frac{4}{5}$, α in quadrant II, $\sin\beta = \frac{-2}{\sqrt{5}}$, β in quadrant III. Find: $\cos\alpha$; $\cos\beta$; $\sin(\alpha+\beta)$; $\cos(\alpha+\beta)$;

b) $\sin\alpha = \frac{5}{13}$, α in quad II, $\tan\beta = -\sqrt{3}$, β in quad II. Find: $\cos\alpha$; $\cos\beta$; $\sin(\beta)$; $\tan(\alpha+\beta)$; $\sin(2\alpha)$;

c) $\sin\theta = \frac{1}{3}$, θ in quadrant II. Find: $\cos(\theta)$; $\sin(\theta+\frac{\pi}{6})$; $\cos(\theta-\frac{\pi}{3})$; $\tan(\theta+\frac{\pi}{4})$

d) Find $\sin(\sin^{-1}\frac{3}{5} + \cos^{-1}(\frac{1}{2}))$ e) Find $\sin(\sin^{-1}\frac{3}{5} - \cos^{-1}(\frac{-4}{5}))$

(19) Solve each equation on the interval $0 \le \theta < 2\pi$

a) $2\sin(\theta) = 1$ b) $2\cos(3\theta) = -\sqrt{3}$ c) $\cos(2\theta) + \cos\theta = 0$

d) $\tan(2\theta) = 1$ e) $\sin(2\theta) = \sin\theta$ f) $\cos(\frac{\theta}{2}) = -1$

g) $4\sin^2\theta = 1$ h) $\tan(3\theta) = \sqrt{3}$ i) $4\cos^2\theta = 3$

20) $\cos\theta = \frac{-3}{5}$, θ in quadrant III. Use double- or half-angle formulas to find

(a) $\sin(2\theta)$ b) $\cos(2\theta)$ $\tan(2\theta)$ d) $\sin(\frac{\theta}{2})$ e) $\cos(\frac{\theta}{2})$

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(29) If $v = 3i - 5j$ and $w = -2i + 3j$

Find a) $2v + 3w$ b) $\|v - w\|$ c) $\|v + w\|$ d) $\|v\| - \|w\|$

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(30) If $\|v\| = 25$, α = angle that v makes with the positive x-axis $= 330°$

Write v in the form $ai + bj$

(21) Identify and graph each polar equation

a) $r = 3\cos(2\theta)$ b) $r = 2\sin(3\theta)$ c) $r = 2\cos(4\theta)$ g) $r = 4 + 3\cos(\theta)$ h) $r = 2 - \cos(\theta)$

d) $r = 2 + 2\cos(\theta)$ e) $r = 3 - 3\sin(\theta)$ f) $r = 4 - 4\cos(\theta)$ i) $r = 3 + 2\sin(\theta)$ j) $r = 4 - \sin(\theta)$

(22) Solve in complex number system the equation

a) $x^2 + 4x + 8 = 0$ b) $x^3 - 8 = 0$ c) $x^4 = 1$

(23) (a) Write in polar form $z = -1 - \sqrt{3}i$ (b) Find and write $(-1 - \sqrt{3}i)^5$ in the standard form $a + bi$

(c) Find the complex cube roots of z

(24) (a) Write in polar form $z = -8 - 8i$ (b) Find and write z^5 in the standard form $a + bi$

(c) Find the complex n*th* roots of z

(25) Vectors $v = 3\mathrm{i} - 5\mathrm{j}$ and $w = -2\mathrm{i} + 3\mathrm{j}$

a) Find $5v + 4w$, $||v||$, $5||v|| + 4||w||$

b) Find the magnitude of v+w

c) Find the angle between v and the x-axis

(26) Solve the triangle with angle $C = 60^o$, and sides $a = 6$, $b = 4$

(27) Find the exact values of a) $\sin(195^o)$ b) $\cos(165^o)$ c) $\tan(195^o)$

d) $\cos(\pi/12)$ e) $\sin(-22.5^o)$ f) $\cos(-22.5^o)$ g) $\sec\frac{15\pi}{8}$

h) $\sin(2\cos^{-1}(4/5))$ i) $\cos(2\sin^{-1}(-2/5))$ j) $\sin 75^o + \sin 15^o$

(28) Establish the identity

a) $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\csc\theta$; b) $\frac{\tan\theta + \cot\theta}{\sec\theta\csc\theta} = 1$

c) $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$; d) $\frac{\sec^2\theta - \tan^2\theta + \tan\theta}{\sec\theta} = \sin\theta + \cos\theta$

e) $\frac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} = \cos(2\theta)$; f) $\cos^4\theta - \sin^4\theta = \cos(2\theta)$

g) $\frac{\sin(3\theta)}{\sin\theta} - \frac{\cos(3\theta)}{\cos\theta} = 2$ g) $\sec(2\theta) = \frac{\sec^2\theta}{2 - \sec^2\theta}$

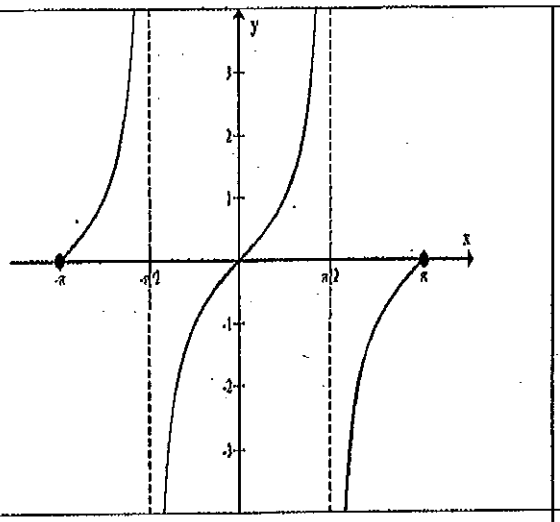
(30) Convert from polar to rectangular $(-4, -\frac{\pi}{4})$, $(6, \frac{7\pi}{6})$

(31) Convert from rectangular to polar $(6, -6)$, $(-3, -3\sqrt{3})$

Review Final Exam (Multiple choice)

(M1) The correct graph of y = cotx is;

A



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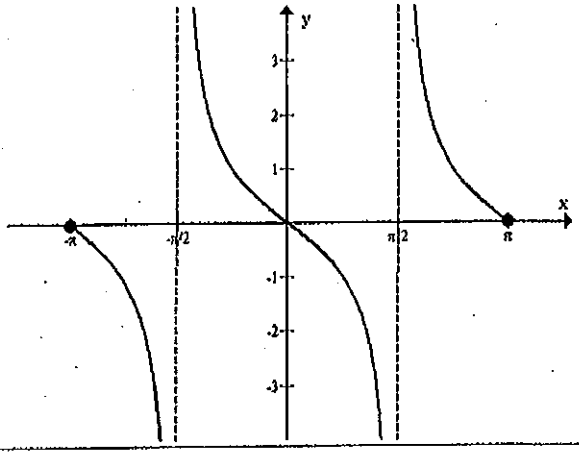

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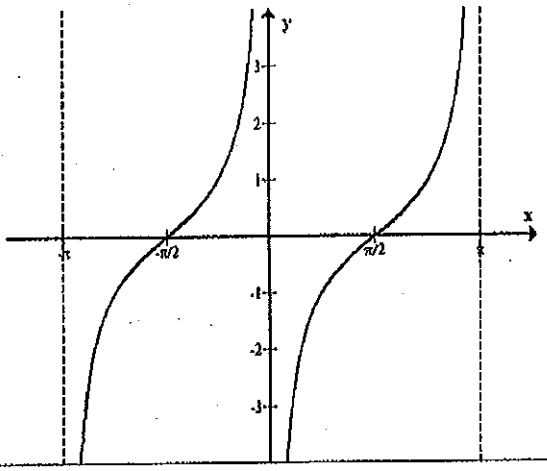

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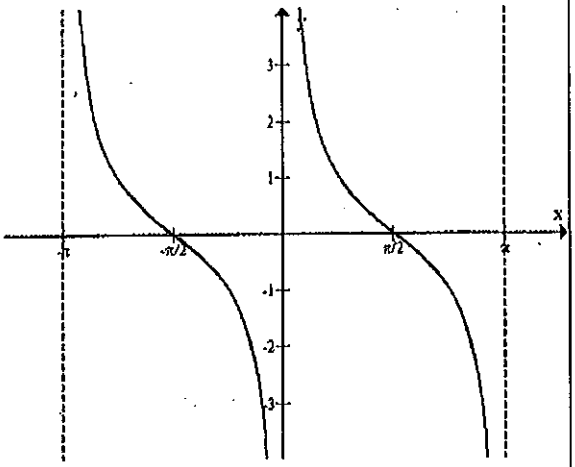

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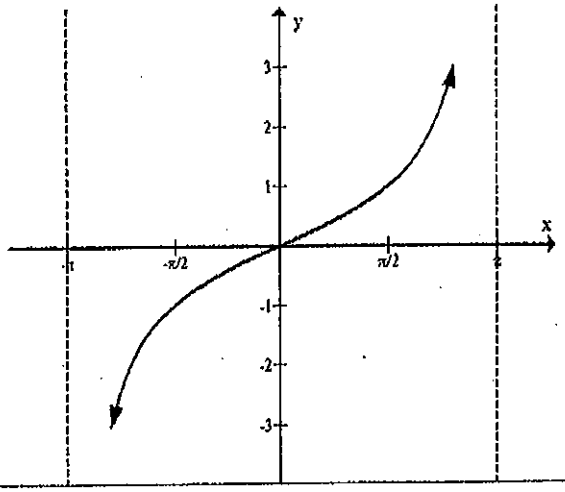

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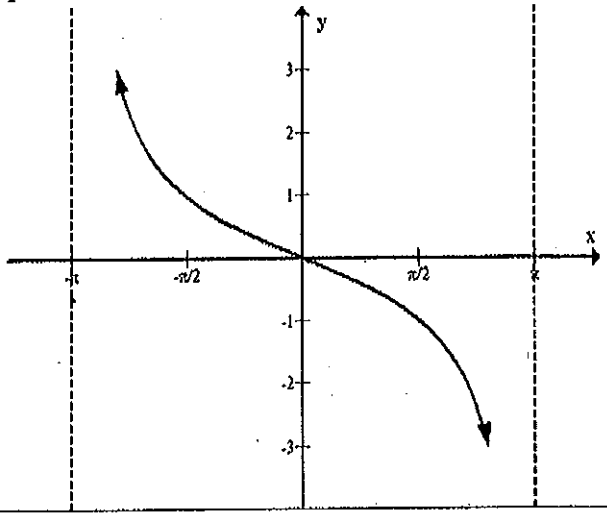
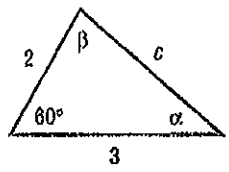


Review MACM4 (multiple choice) (5)

(M2) For the triangle shown, which ONE of the following is true?



A) $c^2 = 2^2 + 3^2$

B) $c^2 = 2^2 + 3^2 + 2(2)(3)\cos 60°$

C) $c^2 = 2^2 + 3^2 - (2)(3)\sin 60°$

D) $c^2 = 2^2 + 3^2 - (2)(3)\cos 60°$

E) $c^2 = 2^2 + 3^2 - 2(2)(3)\cos 60°$

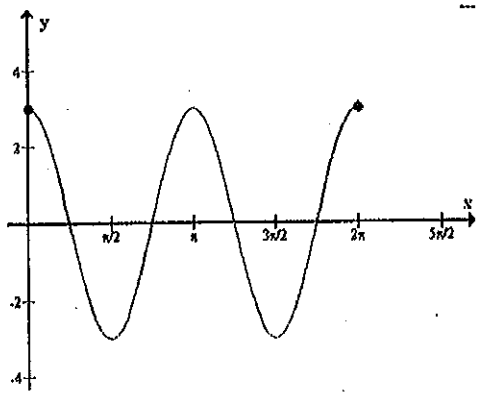
F) $c^2 = 2^2 + 3^2 - 2(2)(3)\sin 60°$

(M3) If $\cos\theta > 0$ and $\cot\theta < 0$, then θ lies in quadrant _____.

A) I B) II C) III D) IV E) Not enough information is given

(M4) Which one of the following is the graph of $f(x) = -3\cos 2x$?

A)



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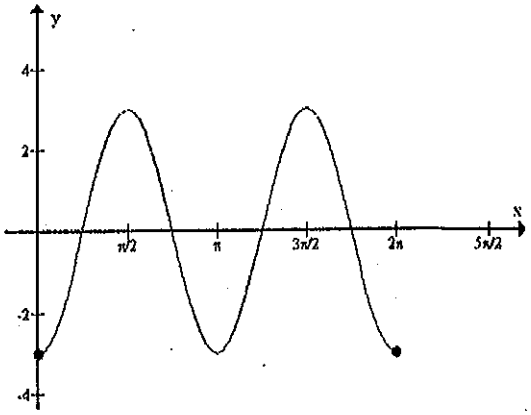

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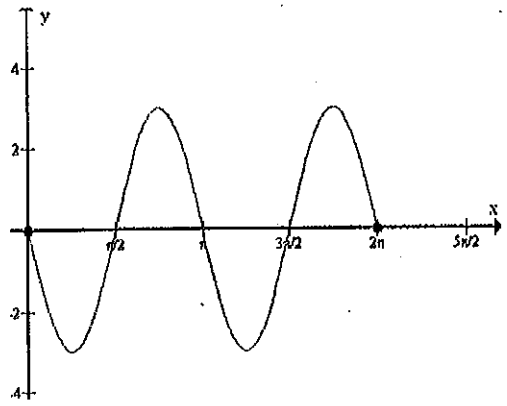
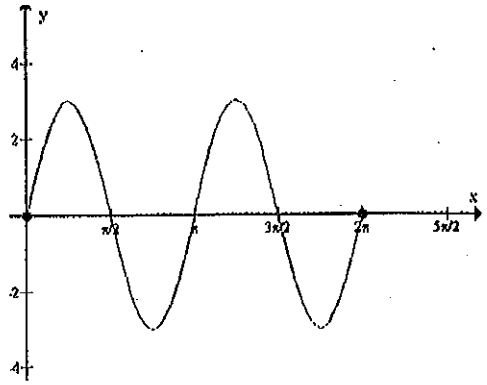


D)



(M5) When graphed in the polar plane, the graph of the equation $\theta = \frac{\pi}{4}$ is a(n)

A) vertical line B) horizontal line C) slanted line D) circle centered at the pole
E) circle not centered at the pole

M6 A 10 ft. ladder leaning against a wall makes a 70° angle with the ground. If h denotes the distance from the top of the ladder to the ground, which equation could we use to find h?

A) $\cos 70^0 = \frac{h}{10}$ B) $\sin 70^0 = \frac{h}{10}$ C) $\sec 70^0 = \frac{h}{10}$ D) $\csc 70^0 = \frac{h}{10}$ E) $\tan 70^0 = \frac{h}{10}$

M7 $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$

A) $\frac{\pi}{6}$ B) $\frac{\pi}{3}$ C) $-\frac{\pi}{6}$ D) $-\frac{\pi}{3}$ E) $\frac{5\pi}{6}$ F) $\frac{2\pi}{3}$

M8 $\sec\frac{5\pi}{4} =$

A) $\frac{1}{\sqrt{2}}$ B) $-\frac{1}{\sqrt{2}}$ C) $\frac{\pi}{4}$ D) $\sqrt{2}$ E) $-\sqrt{2}$ F) 1

M9 Which one of the following is a memorized identity?

A) $\sin 2\theta = 2\sin\theta$ B) $\csc\theta = \frac{1}{\cos\theta}$ C) $\cos^2\theta - \sin^2\theta = 1$

D) $\cos 2\theta = 2\cos^2\theta - 1$ E) $\cos(\alpha+\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ F) none of the above

M10 Convert $\left(3, \frac{3\pi}{4}\right)$ from polar coordinates to rectangular coordinates.

A) $\left(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ B) $\left(\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right)$ C) $\left(-\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right)$ D) $\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

E) none of the above

Non-multiple choice.

Establish that the following equation is an identity.

$(\tan\theta + \cot\theta)\cos\theta = \csc\theta$

Solve the equation $4\sin\theta + 3 = 5$ on the interval $0 \le \theta < 2\pi$.

Multiple Choice, Trig

M11 Use trigonometric identities to find the exact value.

$\frac{\tan 10° + \tan 20°}{1 - \tan 10° \tan 20°} =$

A) $\sqrt{3}$ B) $\frac{\sqrt{3}}{3}$ C) 2 D) $\frac{1}{2}$

M12 Find the given power. Write the answer in standard form.

$(2 - 2i)^5 =$

A) $-64 + 64i$ B) $-\sqrt{2} + \sqrt{2}i$ C) $-128 + 128i$ D) $-64\sqrt{2} + 64\sqrt{2}i$

M13 Find the polar coordinates of $(9, -9)$ for $r > 0$.

A) $\left(9\sqrt{2}, \frac{9\pi}{4}\right)$ B) $\left(9\sqrt{2}, \frac{5\pi}{4}\right)$ C) $\left(-9\sqrt{2}, \frac{7\pi}{4}\right)$ D) $\left(9\sqrt{2}, \frac{7\pi}{4}\right)$

M14 Find the phase shift of the function.

$y = 5 \cos\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

A) 5π units to the right B) $\pi/4$ units to the right

C) $\pi/2$ units to the left D) π units to the left

M15 Solve the equation for solutions in the interval $[0, 2\pi)$.

$2 \cos 2\theta = \sqrt{3}$

A) $\frac{3\pi}{2}$ B) $\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

C) $\frac{\pi}{6}, \frac{11\pi}{6}$ D) $\frac{\pi}{2}$

M16 Find the exact value of the expression under the given conditions.

Find $\sin\frac{\theta}{2}$, given that $\sin\theta = \frac{1}{4}$ and $0 < \theta < \frac{\pi}{2}$.

A) $\frac{\sqrt{10}}{4}$ B) $\frac{\sqrt{8 - 2\sqrt{15}}}{4}$ C) $\frac{\sqrt{6}}{4}$ D) $\frac{\sqrt{8 + 2\sqrt{15}}}{4}$

M17 Solve the equation for the interval $[0, 2\pi)$.

$2 \sin^2 x = \sin x$

A) $x = \frac{\pi}{6}, \frac{5\pi}{6}$ B) $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$ C) $x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$ D) $x = \frac{\pi}{3}, \frac{2\pi}{3}$

M18 Solve the equation in the interval $[0,° 360°)$, to the nearest tenth of a degree.

$\sin^2 x - 8 \sin x + 16 = 0$

A) No solution B) $x = 28.2°, 151.8°, 208.2°, 331.8°$

C) $x = 28.2°, 151.8°$ D) $x = 208.2°, 331.8°$

M19 Find the product. Write answer in standard form.

$z_1 = 8(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6})$ and $z_2 = 3(\cos\frac{\pi}{2} + i \sin\frac{\pi}{2})$

A) $-12 + 12\sqrt{3}i$ B) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$ C) $-1 + \sqrt{3}i$ D) $-\frac{11}{2} + \frac{11\sqrt{3}}{2}i$

Multiple Choice

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Complete the identity.

M20) $\sin^2 x + \sin^2 x \cot^2 x = ?$

A) $\cot^2 x - 1$ B) 1 C) $\sin^2 x + 1$ D) $\cot^2 x + 1$

M21) $\dfrac{\sin(3\theta) + \sin(9\theta)}{\cos(3\theta) + \cos(9\theta)} = ?$

A) $\tan(6\theta)$ B) $2\tan(6\theta)\tan(3\theta)$ C) $\tan(3\theta) + \tan(9\theta)$ D) $\tan(6\theta)\cot(3\theta)$

M22) $\dfrac{\sin\theta}{1+\sin\theta} - \dfrac{\sin\theta}{1-\sin\theta} = ?$

A) $\sin\theta \tan\theta$ B) $\sec\theta \csc\theta$ C) $-2\tan^2\theta$ D) $1 + \cot\theta$

M23) $\cos(\alpha+\beta)\cos(\alpha-\beta) = ?$

A) $\cos^2\beta - \sin^2\alpha$ B) $\cos^2\beta - 2\sin^2\alpha \sin^2\beta$

C) $2 - \sin^2\alpha - \sin^2\beta$ D) $\cos(\alpha^2)\cos(\beta^2) + \sin(\alpha^2)\sin(\beta^2)$

Convert the polar equation to a rectangular equation.

M24) $r = 5\cos\theta + 4\sin\theta$

A) $x^2 - y^2 = 5x + 4y$ B) $5x + 4y = 0$ C) $x^2 + y^2 = 5x + 4y$ D) $x^2 + y^2 = 4x + 5y$

Use the fact that the trigonometric functions are periodic to find the exact value of the expression.

M25) $\csc 960°$

A) $-\dfrac{1}{2}$ B) $-\sqrt{3}$ C) $-\sqrt{2}$ D) $-\dfrac{2\sqrt{3}}{3}$

Find the quotient. Write answer in standard form.

M26) $\dfrac{8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}{3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)}$

A) $1 + \sqrt{3}i$ B) $\dfrac{4}{3} + \dfrac{4\sqrt{3}}{3}i$ C) $8 + 8\sqrt{3}i$ D) $\dfrac{5}{2} + \dfrac{5\sqrt{3}}{2}i$

Express the sum or difference as a product of sines and/or cosines.

M27) $\sin(4\theta) - \sin(6\theta)$

A) $-2\sin\theta\cos(5\theta)$ B) $2\sin(5\theta)\cos\theta$ C) $2\cos(4\theta)\cos(5\theta)$ D) $-2\sin\theta$

Write the trigonometric expression as an algebraic expression containing u and v.

M28) $\cos(\sin^{-1}u - \cos^{-1}v)$

A) $v\sqrt{1-u^2} - u\sqrt{1-v^2}$ B) $uv + (\sqrt{1-u^2})(\sqrt{1-v^2})$

C) $uv - (\sqrt{1-u^2})(\sqrt{1-v^2})$ D) $v\sqrt{1-u^2} + u\sqrt{1-v^2}$

1 in

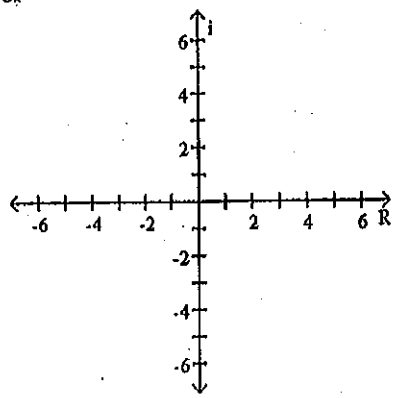
Multiple choice Review Final MAC1114 (9)

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Plot the complex number in the complex plane.

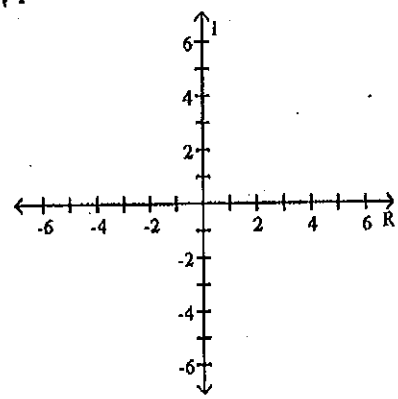
(M29) $4+6i$

34) _____



(M30) $-6+i$

35) _____



MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write the complex number in rectangular form.

(M31) $8\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ read $8\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

36) _____

A) $4\sqrt{3}+4i$ B) $\frac{1}{4}+\frac{\sqrt{3}}{4}i$ C) $4+4\sqrt{3}i$ D) $\frac{\sqrt{3}}{4}+\frac{1}{4}i$

(M32) $4(\cos 300° + i \sin 300°)$

37) _____

A) $-2+2\sqrt{3}i$ B) $2-2\sqrt{3}i$ C) $-2\sqrt{3}-2i$ D) $2\sqrt{3}-2i$

(M33) $9(\cos 180° + i \sin 180°)$

38) _____

A) $9i$ B) 9 C) -9 D) $-9i$

(M34) **Write the complex number in polar form. Express the argument in degrees, rounded to the nearest tenth, if necessary.**

$\sqrt{3}+i$

39) _____

A) $4(\cos 30° + i \sin 30°)$ B) $2(\cos 60° + i \sin 60°)$

C) $2(\cos 30° + i \sin 30°)$ D) $4(\cos 60° + i \sin 60°)$